Physics 54

Nuclear Physics

> *It's very difficult to make predictions—especially about the future.*
> *—Niels Bohr*

Overview

The discovery of radioactivity in 1896 caused a shock among chemists, because it suggested that the atom is not the eternal, immutable object they assumed it to be. This became even clearer within a few years, when radioactive process were studied carefully and it was shown that, in some of them, existing elements in the sample under study disappeared and were replaced by others that had not previously been there.

Soon three kinds of "rays", called α, β and γ, were identified among the emissions given off in radioactive "decays". The α and β rays, which could be deflected by strong magnetic fields, were determined to be charged particles, while γ rays were found to electrically neutral. After much further work it was established that:

- α rays have charge +2e and mass 4 times that of hydrogen.
- β rays are electrons (mostly — some turn out to be positrons).
- γ rays are photons of exceptionally high frequency.

Since charge is always conserved, emission of α or β rays must be accompanied by a change in the charge of the nucleus. That is, a transmutation takes place, changing atoms of one element into those of another. (This does not occur in emission of the γ rays, in which the nucleus is undergoing a transition from a higher to a lower energy state.) After helium had been discovered on earth, it was found that α rays quickly become neutral helium atoms. In fact, nearly all of the helium we find in the earth's crust (in natural gas) comes from nuclear decays giving off α rays.

It was by using α rays from a strong radium source that Rutherford was able to perform many of his studies, including the ones that revealed the existence of the atomic nucleus. Subsequently he was also able to make a nucleus absorb an α ray and become a different kind of nucleus: the first man-made nuclear transmutation.

It was realized early on that the nucleus of hydrogen is an “elementary” particle, and it was named the **proton**. The other subatomic particle known at that time was the electron, and for about a decade it was assumed that these are the only ones.

They are also the only common microscopic particles that are absolutely stable against decay.

For all nuclei except hydrogen the mass is a larger multiple of the proton mass than the charge is of the proton charge, so the nucleus cannot consists only of protons. This was explained by assuming that there are also electrons in the nucleus, whose negative charge cancels the charge of some of the protons, but whose mass is negligible. Helium, for example, would have a nucleus consisting of four protons and two electrons.

This model found less agreement with the facts as more was learned about the various nuclei in the 1920’s. A more successful model assumed the existence of another “elementary” particle with a mass (nearly) that of the proton, but with zero charge. This particle, already named the **neutron**, was discovered experimentally in 1932. From that point on, the nucleus has been thought of as consisting of protons and neutrons (which are referred to generically as *nucleons*).

The existence of even temporarily stable nuclei (except hydrogen) presents a puzzle. Protons repel each other through the Coulomb force, which at distances of nuclear size ($\sim 10^{-15}$ m) is very strong. What holds the nucleus together? Clearly some other force is at work, and it must be a strong one. (It was soon named the “strong interaction.”) But it also must be of exceptionally short range, since nuclei in adjacent atoms in a solid do not seem to interact with each other at all through this force — fortunately, for the stability of matter. From the 1930’s to the present day, research into the nature of this interaction has occupied the attention of generations of physicists. We can say with certainty that there is no simple formula for it like Coulomb’s law.

More recent models assume nucleons to be composed of more truly “elementary” objects called *quarks*, bound together (three at a time) by the influence of other objects called *gluons*. The gluon force accounts for the strong interaction between nucleons as well. There is reasonably compelling experimental evidence for this model. The force produced by gluons has the strange property of becoming *stronger* with increasing distance (a bit like a very stiff spring) and this prevents quarks from ever breaking their bonds to other quarks, which is why we do not observe isolated quarks.

Even without a detailed theory of the strong interaction, there are useful things to be said about the nuclei and their stability. We will discuss some of them. We will also look at how radioactive decays are described.

Binding energy

One striking aspect of the strong interaction is its “charge-independence”, which means that the strong force between two protons, two neutrons, or a proton and a neutron all have about the same strength and are all attractive. Since two protons repel each other by the Coulomb force and therefore raise the potential energy, it is advantageous in creating a bound system to have more neutrons than protons. For this reason, all the stable nuclei except the very lightest ones have more neutrons than protons.

However, both protons and neutrons are “fermions” (like electrons), meaning that they obey the Pauli exclusion principle which forbids any two *identical* fermions to exist in exactly the same state. Adding more neutrons thus often means *increasing* the energy, since the new ones must go to states of higher energy (like electrons in an atom). This reduces the binding. There is a limit to the advantage of neutrons over protons.

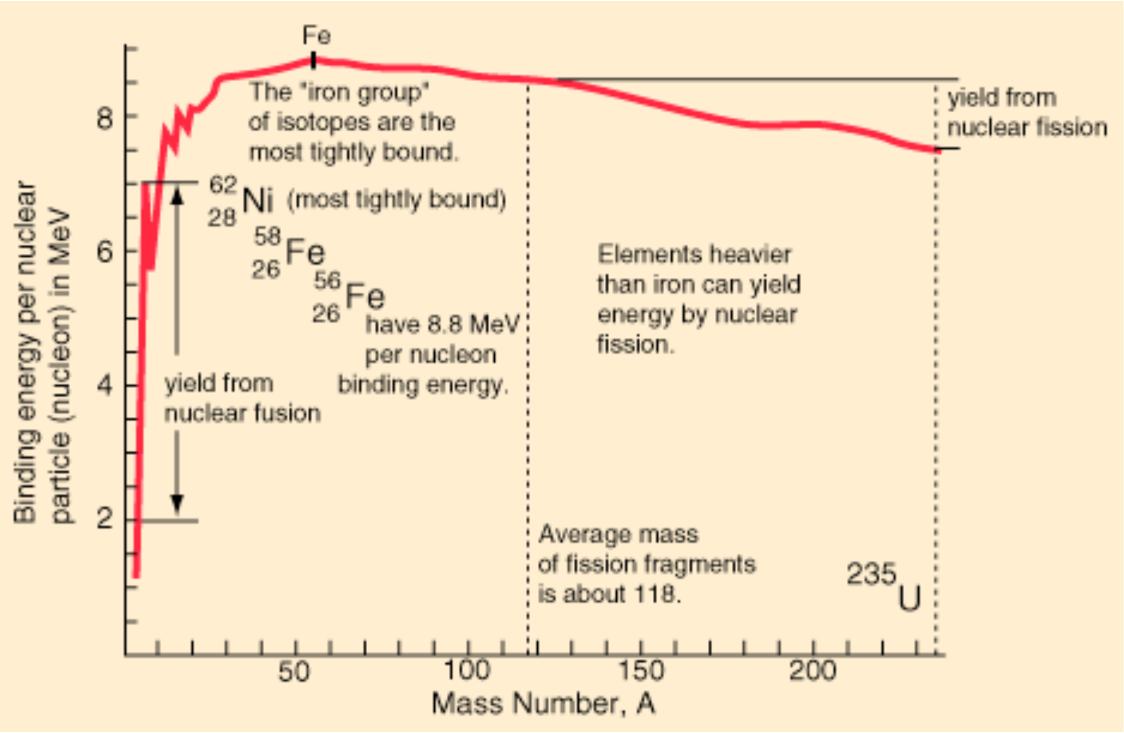
The total strength of the binding is given by the **binding energy**, E_b, which is the energy required as input to pull all the protons and neutrons apart and move them to large distance from each other. This can easily be related to the total mass of the nucleus through Einstein’s famous $E = mc^2$ formula for the rest energy of an object:

Binding energy formula	$^{A}_{Z}Mc^2 = [Zm_p + (A-Z)m_n] \cdot c^2 - E_b$

Here $^{A}_{Z}M$ is the mass of a nucleus that has A total nucleons, consisting of Z protons and $(A-Z)$ neutrons, m_p is the proton mass and m_n is the neutron mass. The actual rest energy of the nucleus is lower than the total rest energy of its constituents by the amount of the binding energy.

The integer A is called the ”mass number” or “nucleon number”. A fundamental law of nature says that in any nuclear reaction A must be conserved.

The plot of binding energy per particle vs. A for stable nuclei is shown:



There are some general trends worth emphasizing in this plot:

- For A below about 50, adding new particles increases the binding energy per particle, often leading to a new stable nucleus. Indeed, fusing two very light nuclei can result in release of a great amount of energy; it is the source of energy in stars, including our sun.

  And, of course, of hydrogen bombs. It is also the hoped for energy source of the future, controlled fusion of light nuclei.

- Adding single nucleons results in creation of heavier elements, until the iron group is reached. Beyond that, adding another nucleon results in an unstable nucleus. The process of element building in ordinary stars thus must stop with the iron group. Heavier elements must have been formed under very extreme conditions, such as during a supernova explosion.

  Because such elements are abundant in the earth and other planets, this means that our solar system must have accumulated from the debris of a supernova about 5 billion years ago.

- For large values of A many nuclei are unstable because it releases energy for them to emit something massive, like a nucleon or an α particle, or even to break up by fission into large fragments of roughly equal mass.

  The fission of certain heavy nuclei provides the energy for today's nuclear power plants, and was the energy source for the nuclear bombs used in WW II.

The general shape of the above curve can be understood qualitatively in terms of several competing effects:

- The Coulomb repulsion of the protons, proportional to the number of proton *pairs* and hence roughly to Z^2. This reduces the binding energy for elements with large Z, resulting in the downward slope of the curve for large A.
- The Pauli principle, limiting the usefulness of excess neutrons in "gluing" the protons together.
- The fact that in the nucleus, as in a liquid, only nearest neighbors interact, and at the surface there are nearest neighbors only on one side.

A "semi-empirical" formula involving these effects gives a reasonably good account of the binding energy curve.

The average binding energy per particle ranges from about 4 to 8 MeV. This is very large compared to the binding energies (of order 1 eV) of electrons in atoms or molecules. As a result, the energies released in fusion or fission processes are very large compared to energy releases in chemical reactions.

Instability and decays

The systematic analysis of nuclear decay rates treats a system containing a large number N of identical nuclei, and makes the following assumptions:

- Each decay is an individual event involving only the decaying nucleus, and is independent of all the others.
- The probability that a given nucleus will decay within unit time is the same for all the identical nuclei. This probability per unit time is denoted by λ.

In time dt the *decrease* in the number N of "parent" nuclei, N, is given by

$$dN = -N\lambda\, dt$$

Writing this as $dN / N = -\lambda\, dt$, we integrate and find the number of parent nuclei left after time t:

Radioactive decay formula	$N(t) = N_0 e^{-\lambda t}$

Here N_0 is the number of parent nuclei at $t = 0$.

The average lifetime of a parent nucleus before decay is $\tau = 1/\lambda$. It is also common practice to cite the time it takes for half of the parent nuclei to decay:

$$\text{Half-life: } \tau_{1/2} = \ln 2 / \lambda \approx 0.693 / \lambda$$

The list of unstable nuclei exhibits a vast range of lifetimes, from small fractions of a second to 10^{15} yr. "Natural" radioactivity results from decays of the long-lived nuclei and their "daughters". Numerous radioactive nuclei of relatively short lifetimes (which therefore do not occur abundantly in nature) have been produced artificially by bombarding stable nuclei with protons, neutrons, or other projectiles. Many of the radioactive substances used in medical practice are made this way.

Like an atom, a nucleus can exist not only in its lowest (ground) states but also in various excited states with higher energy. When the nucleus returns to the ground state by emission of a photon, the result is a γ ray. The frequency of the photon is determined by the difference between the energy levels, as in an atom, according to $\Delta E = hf$. Since the differences in energy between states of a nucleus are typically a few MeV, these photons are like "hard" X-rays and can deposit considerable energy in absorbing materials; this makes them dangerous to living organisms.

Other decays of nuclei involve changes of the proton number Z, or the mass number A, or both.

The very heavy nuclei (heavier than lead) often decay by emission of an α particle, which reduces the number of protons (Z) by 2 and the nucleon number (A) by 4. The theoretical description of this process involves a strange quantum phenomenon called "tunneling". The original nucleus is regarded as consisting of the final nucleus with an α particle bound inside it. The α particle cannot get out by acquiring enough energy to surmount the potential energy barrier that binds it. Instead it is believed to "tunnel" through the barrier, because (according to quantum mechanics) it has a small but non-zero probability of simply being found outside. This probability is very small (and decreases rapidly as the binding of the α particle increases) so the average lifetime of some of these nuclear species is very long, longer than the age of the earth and perhaps of the universe.

The "daughter" nucleus resulting from α particle emission may be either stable or unstable. There is often a series of decay sequences, each decay having its own daughter and its own characteristic average lifetime.

Because α particles are charged and massive, moving relatively slowly, they are not dangerous from outside the body — the skin stops them. Only if they are released inside the body, e.g., by a radioactive gas in the lungs, are they likely to pose a health hazard. Radon is such a gas; it is a product of decay of uranium, which is plentiful in rocks and some soils. Radon accumulations in closed spaces can be dangerous.

In ordinary β decay, an electron is created and emitted from the nucleus, along with a very light particle called the neutrino, which has no charge and possibly (like the photon) no mass. From the point of view of the nucleus, β decay amounts to conversion of a neutron into a proton. (Indeed, a "free" neutron is unstable and decays into a proton just this way.) This process involves creation of new particles, and is described in terms of another fundamental interaction. Because the probability per unit time of the process is relatively small, the interaction responsible is called "weak".

In current theory, the weak and electromagnetic interactions are regarded as different manifestations of the same thing, called the electro-weak interaction.

In some cases the reverse happens, in which an electron among those most tightly bound in the atom is "captured" by the nucleus, converting a proton into a neutron. Yet another possibility is emission of a positron (the antiparticle of the electron, having charge $+e$) while a proton becomes a neutron. These processes are also results of the weak interaction.

The energies released in β decay vary widely, but are substantial enough that the particles can penetrate the skin and cause cell damage. Indeed, some cancer therapies use this form of radiation to destroy cancer cells.

In order for any of these decays to take place, three general principles *must* be obeyed:

- Conservation of energy. This requires that the mass of the parent nucleus be greater than the mass of the daughter plus the masses of any decay products, because the kinetic energy of the final particles must be positive. The difference between the initial and final total masses, multiplied by c^2, gives the amount of energy released as kinetic energy of final particles and energy of photons. This energy released is called the Q-value.
- Conservation of linear and angular momentum. These apply because the system is assumed to be isolated when it decays (no external forces or torques).
- Conservation of charge. The initial charge must equal the total charge of all the final particles.
- Conservation of nucleon number. (This is a special case of a more general law called conservation of “baryon” number.) The total number of protons and neutrons after the process must be the same as before.

A process that violates any of these basic laws will not occur.

There are more specific restrictions called “selection rules” that limit the possibilities further, or else make the probabilities small and the lifetimes correspondingly longer.